# 2P12 A detection technique for signals hidden in noises: *the Lock-in amplifier and the modulation approaches*

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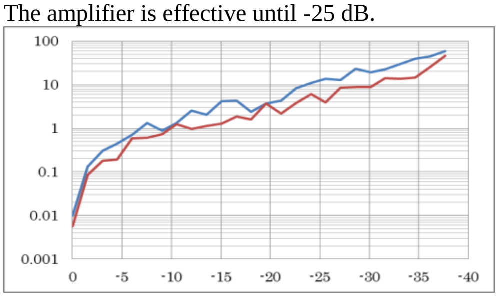
### *Objective:*

We discuss a technique to detect signals hidden in noises. We consider one observation, and concerned with cases that statistical approaches are difficult.

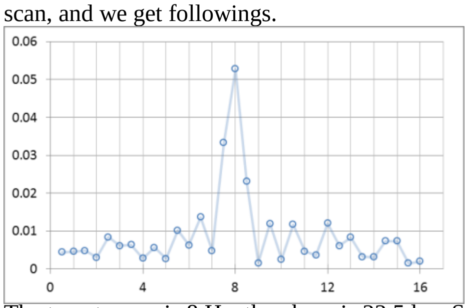
### *Lock-in amplifier:*

A signal is defined, $A*\sin(nt+\varphi)$, (1)
where A, n, φ are the amplitude, frequency, phase.
If reference waves are operated, we get,
$A*\sin(nt+\varphi)*\sin(nt)=(A/2)\{\cos(2nt+\varphi)+\cos(\varphi)\}$, (2)
$A*\sin(nt+\varphi)*\cos(nt)=(A/2)\{\sin(2nt+\varphi)-\sin(\varphi)\}$. (3)
The DC term is invariable for t in (2); thus, if the noise is uniform, the DC and phase are detected as,
$(A/2)*\{\cos^2(\varphi)+\sin^2(\varphi)\}^{0.5}$, $\tan(\varphi)=\sin(\varphi)/\cos(\varphi)$. (4)
Because; $\Sigma t\rightarrow\infty[CC*Rd(t)+C*A*\sin(nt+\varphi)]*\sin(nt)$
$=C*(A/2)*\cos(\varphi)$, (5)
$C+CC=1$, *S/N ratio:* $dB=-20*\log10(C)$, (6)
Error ratio in case of 1 times measurement, data number is 10k. Horizontal scale is S/N ratio, [dB].
Blue is error [%] of the amplitude, and red is the phase.
The amplifier is effective until -25 dB.




### *Frequency scan:*

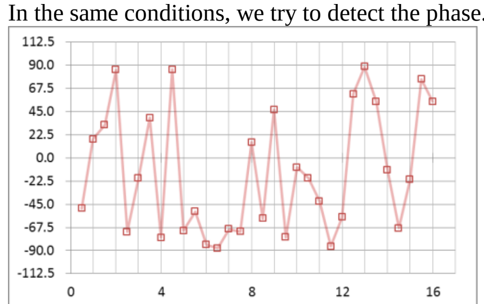
To detect frequency of the target wave, we consider a scan for reference waves. Including half periods into the scan, and we get followings.




The target wave is 8 Hz, the phase is 22.5deg, S/N is -20dB, and data are 10k. The horizontal axis of this figure is the scan frequency [Hz]. The vertical is relative DC intensity. In the level of -20dB, the frequency is detected.

### *Phase detection:*

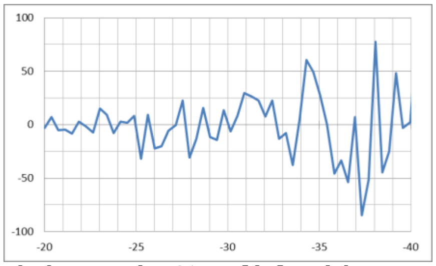
In the same conditions, we try to detect the phase.




The horizontal is the scan frequency [Hz]. The vertical is phase [deg]. The correct phase [deg] is got only in referencing true frequency (8Hz in the figure). Even if there is small error in the input waves, uncertain phase is got.

### *Chopper Modulation:*

We define a chopper in ***discrete*** time,
$CP(t+1)=CP(t)$, (7)
Using the chopper, an integration of sine is,
$I=\int_0^{\pi}\sin(t)dt=2$, $J=\int_0^{\pi}\sin(t)CP(t)dt\sim0$, and,
$K=\int_0^{\pi}Rd(t)dt\sim0$, $L=\int_0^{\pi}Rd(t)CP(t)dt\sim0$, (8)
It is necessary that integration is significant;
$|I| > \max\{|K|,|L|,|J|\}$. (9)
We consider 2 kinds of choppers;
$I=\int_0^{\pi}\{C*\sin(t)+CC*Rd(t)\}dt$
$-\int_{\pi}^{2\pi}\{C*\sin(t)+CC*Rd(t)\}dt$,
$J=\int_0^{\pi}\{C*\sin(t)+CC*Rd(t)\}*CP(t)dt$
$-\int_{\pi}^{2\pi}\{C*\sin(t)+CC*Rd(t)\}*CP(t)dt$, (10)
$Err=(4C-I)/(4C)$, {4 is expectation}. (11)
The necessary condition is, $|I| > |J|$. (12)
In case of 10k data, we get next figure.

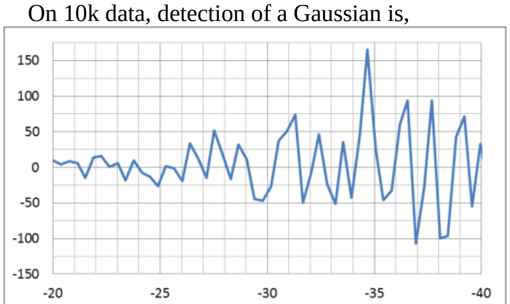



The horizontal is S/N in [dB], and the vertical is error %. The approach is effective until -33dB. The necessary is until -37dB. ***The approach doesn't require special relation of triangular functions***, and a numerical integration is necessary only; thus, any hidden-function can be detected.

### *Detection of a Gaussian:*

We consider a Gaussian in [0,1]. It is defined as $G(x; \alpha,m)=\exp\{-\alpha(x-m)^2\}$, $\alpha>0$, $G()>0$ that has an expectation, $\int\pm\infty G(x; \alpha)dx=(\pi/\alpha)^{0.5}$. We adopt $\alpha=20$, $m=0.5$. The Gaussian and uniform random number are mixing of coefficient C in (10).

On 10k data, detection of a Gaussian is,

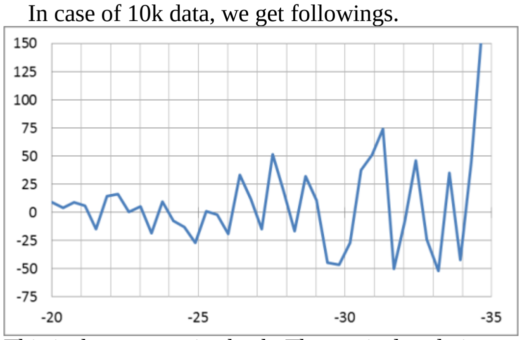



The precision of numerical integration is 0.16% for [0,1]. In the figure, the horizontal is S/N in [dB], and the vertical is error %. The effective S/N is -24dB, and the necessary condition is -31dB.

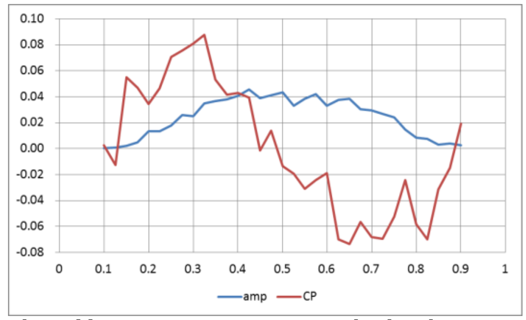
### *Symmetric condition:*

We discuss symmetric condition for the mixing of Gaussian and random numbers. The characters are;
1) for inversion of constant C, it is symmetric.
2) at x=m, it is mirror symmetric, G(x)=G(-x).

In case of 10k data, we get followings.




This is the symmetric check. The vertical scale is error %, and the horizontal is S/N [dB]. Symmetry holds until -26dB.




The red line is a mirror symmetric check, whose algorithm is;

$CP(t)=1$ $(0<\theta-w<t<\theta)$,

$CP(t)=-1$ $(\theta<t<\theta+w<1)$, $w>0$, (13)

$IG=\int_{0,1}\{C*G(t;\alpha,m)+CC*Rd(t)\}*CP(t)dt/w$. (14)

The $\theta$ is scanned, and at $IG(t)\sim 0$, a "t" is detected. The t corresponds with "m". This is an important condition. When C=-20dB, we get that calculated {m, C} are {0.46, 0.0398}, and the expectation is {0.50, 0.0396}. In case of C=-26dB, the "m" is uncertain between [0.45, 0.5]; however, on 100k data, it converges to 0.488. The condition is weaker against noises than that of the symmetry.

### *Sufficient check based on Gaussian product:*

Gaussian product is a Gaussian; i.e.;

$A_1\exp\{-\alpha_1(x-c_1)^2\}*A_2\exp\{-\alpha_2(x-c_2)^2\}$

$=A_3\exp\{-\alpha_3(x-c_3)^2\}$, (15)

$u1=0.5/\alpha_1$, $u2=0.5/\alpha_2$,

$R=1.0/(u1+u2)$, $v=u1u2R$,

$B=(u2c_1+u1c_2)R$, $D=(u2c_1^2+u1c_2^2)R$,

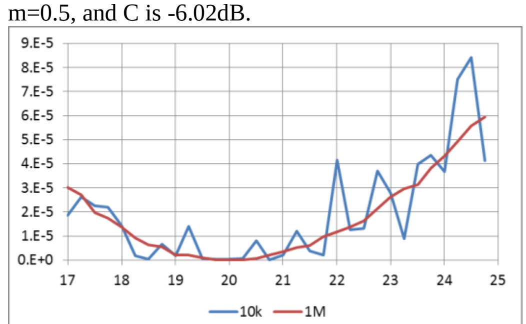
$\alpha_3=0.5/v$, $A_3=A_1A_2\exp\{(B^2-D)\alpha_3\}$. (16)

Thus; by using (17), $\beta$ can be scanned.

$IH=\int_{0,1}\{C*G(t;\alpha,m)+CC*Rd(t)\}*G(t;\beta,m)dt$, (17)

Where, $C=A_1$, $A_2=1$, $c_1=c_2=m$.

We get followings for 10k and 1M data. The $\alpha_1$ is 20, m=0.5, and C is -6.02dB.




The vertical is $\{(\pi/\alpha_3)^{0.5}-IH\}^2$, and the horizontal is $\beta$. Certainly, the exponent of hidden Gaussian is detected. The scanning hasn't good S/N.

### *Conclusion:*

We discuss a modulation of Lock-in amplifier, and show an approach to detect a Gaussian hidden in noises. The approach is effective for one measurement.